A NOTE ON THE ARTICLE "SOME EXPERIMENTAL n-PERSON GAMES"[1]

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§1. INTRODUCTION

The purpose of this note is to present a different, and I feel revealing, analysis of some of the data reported by Kalisch, Milnor, Nash, and Nering in Decision Processes [1]. I shall assume that the reader is familiar with their paper, and so it will be necessary to review only a few aspects of this work. First, while several different experiments are reported, only one of these was devoted to cooperative games with unrestricted side-payments. In that case, two four-person constant-sum games, each in two S-equivalent forms, were each run 16 times -- eight runs per S-equivalent form, one five-person constant-sum game was run three times, and one seven-person constant-sum game was run twice. Only the four-person games were sufficiently free of experimental artifacts, e. g., the effects of the seating arrangements of the subjects, and were repeated sufficiently often for our purposes, so we shall restrict our attention to those cases. Second, the subjects were told what each possible coalition would receive, i.e., they were given the characteristic function directly, and they were left to their own devices for 10 minutes to form coalitions and to agree upon a division of the spoils. Their agreements were supposed to be reported to an umpire, and he enforced them. Evidently, the subjects did not report all agreements on the division of payments, but this did not result in disputes. Third, the authors compared their experimental results with the several equilibrium theories existing at the time and with the basic assumption of n-person theory that S-equivalent games should receive the same strategic considerations. With respect to the major theory of n-person games -- the theory of solutions due to von Neumann and Morgenstern [5] -- they were forced to the conclusion that "It is extremely difficult

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to tell whether or not the observed results corroborate the von Neumann-Morgenstern theory. This is partly so because it is not quite clear what the theory asserts." (p. 313.) The average observed imputation was compared with the Shapley value [6] and they found that "There seems to be a reasonably good fit between the observed data and the Shapley value, considering the small number of plays which are being averaged. There is, however, a tendency for the actual outcome to be more extreme than the Shapley value." (p. 309.) Milnor [4] has offered three different definitions each limiting the admissible outcomes in some more or less intuitively acceptable way. With respect to the data, two appear not to be very relevant, but one is. If $b(i)$ denotes the maximal incremental contribution of player i to the coalitions of which he is a member, Milnor argues that player i's outcome should not exceed $b(i)$. "It appears that $b(i)$ is usually compatible with the outcomes, in particular in the case of the four-person games (a deviation in one of the cases was subsequently related to the statement made by one of the subjects after the experiment was over that he had made a mistake in reasoning). The agreement is less favorable in the case of the five-person game -- some players getting more than their "maximum share" $b(i)$ -- a circumstance which may be related to the fact that the players rushed into coalitions, splitting the payoff evenly, without really studying the strategic possibilities." (p. 315.) Another important comparison, though not directly of an equilibrium nature, was made with the concept of strategic equivalence. They compared the average imputations of the two pairs of S-equivalent games, and they reached the conclusion that S-equivalent games appear not to receive similar treatment. They argue that "the fact that coalitions with high characteristic function are most likely to form, the tendency of members of a coalition to split evenly, and the non-linearity of the utility function all tend to disrupt the concept of strategic equivalence." (p. 313.)

In this note, their data -- fortunately they gave tables of raw data -- will be compared with an equilibrium theory which was not in existence at the time of the experiment. The comparison is interesting in its own right, and it suggests that with regard to the assumption that S-equivalent games shall receive similar strategic considerations a somewhat more subtle interpretation of the data is required than that sketched above. Indeed, one is almost able to argue that the assumption is confirmed.

§2. Ψ-STABILITY

Elsewhere [2, 3], I have offered an equilibrium notion called Ψ-stability. Since it will be assumed that the reader is familiar with at least one of these papers, the central concepts can be recapitulated without presenting any intuitive defense. First, it is assumed that an outcome of a game is not completely characterized by an imputation X, but rather a pair (X, τ) is required, where τ is a partition of the players into coalitions. A partition τ is called a coalition structure. Second, there is assumed to exist a function Ψ with domain the set of coalition structures and range the class of all sets of subsets of the players. It is further assumed that if $T \in \tau$, then $T \in \Psi(\tau)$. S is interpreted as a possible change from the coalition structure τ if and only if $S \in \Psi(\tau)$. Third, if v denotes the $0, 1$ normalized characteristic function of a game, then a pair (X, τ) is said to be Ψ-stable if no admissible change insures profit to the participants in the change, i.e.,

i) for every $S \in \Psi(\tau)$, $v(S) \leq \Sigma_{i \in S}\, x_i$,

and any member of a non-trivial coalition receives more than he could insure himself when playing in isolation under the most adverse conditions, i.e.,

ii) $x_i = 0$ implies $\{i\} \in \tau$.

The crux of any application of this definition is the determination of the function Ψ, which summarizes the "sociological" limitations on changes in collusive arrangements. The authors' general discussion of the subjects' behavior suggests a likely candidate for Ψ. "Coalitions of more than two persons seldom formed except by being built up from smaller coalitions. Further coalition forming was usually also a matter of bargaining between two groups rather than more." (p. 306.) While they are not explicit on this point, we may also suppose that if a player demanded so much of a coalition that they would be better off without him, they threatened him with expulsion. Thus, the function Ψ suggested is characterized by admitting all coalitions in τ, the union of any two coalitions in τ, and the expulsion of any single player from a coalition in τ, i.e.,

$$\Psi(\tau) = [S \mid \text{either there is an } i \text{ such that } S \cup \{i\} \in \tau \text{ or there exist } T \text{ and } T' \in \tau \text{ such that } S = T \cup T'].$$

§3. COMPARISON WITH THE DATA

Tables I and II present the data reported by Kalisch et al in slightly modified form: first, they have been regrouped according to the equilibrium coalition structure, and second, they have all been reduced to $0, 1$ normalization. The characteristic functions of the two games are given in the captions by giving their values for three two-element coalitions -- this is sufficient because the games are constant-sum and normalized. The exact form in which they were presented to the subjects may be found in the report of the experiment. In the statement of the

TABLE I

Comparison between data from RAND experiment and Ψ-stability theory. Symmetric 4-person constant-sum game in 0, 1 normalization: $v(\{1, 2\}) = v(\{1, 3\}) = v(\{1, 4\}) = 1/2$. Data are reduced to 0, 1 normalization; the round-off error is 0.01. The function Ψ is described in text. In all cases the lower limit on x_i is omitted as it is always confirmed.

Coalition Structure and corresponding Ψ-stable imputations	Game No.	Run No.	Observed Imputation Players 1	2	3	4	Incompatibilities between theory and data
$[\{1\}, \{2, 3, 4\}]$	2	7	.00	.40	.51	.09	None
$x_2 + x_3 \geq .50$	2	8	.00	.30	.43	.28	None
$x_2 + x_4 \geq .50$	3	2	.00	.26	.36	.38	None
$x_3 + x_4 \geq .50$	3	6	.00	.36	.28	.36	None
$x_1 = .00$							
$[\{3\}, \{1, 2, 4\}]$							
$x_1 + x_2 \geq .50$	3	4	.38	.36	.00	.26	None
$x_1 + x_4 \geq .50$							
$x_2 + x_4 \geq .50$							
$x_3 = .00$							
$[\{1, 2, 3\}, \{4\}]$							
$x_1 + x_2 \geq .50$	2	2	.48	.20	.33	.00	None
$x_1 + x_3 \geq .50$	2	4	.25	.31	.44	.00	None
$x_2 + x_3 \geq .50$	3	3	.34	.33	.34	.00	None
$x_4 = .00$							
$[\{1, 4\}, \{2, 3\}]$	2	1	.45	.13	.38	.05	None
$x_1 + x_4 = .50$							
$x_2 + x_3 = .50$	2	3	.19	.19	.31	.31	None
	2	5	.21	.19	.31	.29	None
	2	6	.28	.19	.31	.23	None
$[\{1, 2\}, \{3, 4\}]$	3	1	.25	.25	.25	.25	None
$x_1 + x_2 = .50$	3	7	.25	.25	.25	.25	None
$x_3 + x_4 = .50$							
$[\{1, 3\}, \{2, 4\}]$							
$x_1 + x_3 = .50$	3	5	.25	.25	.25	.25	None
$x_2 + x_4 = .50$							
$[\{1, 2, 3, 4\}]$	3	8	.25	.25	.25	.25	Total (see text)
Not stable							

TABLE II

Comparison between data from RAND experiment and Ψ-stability theory. Non-symmetric 4-person constant-sum game in 0, 1 normalization: $v(\{1, 2\}) = 3/4$, $v(\{1, 3\}) = 1/2$, $v(\{1, 4\}) = 1/4$. Data are reduced to 0, 1 normalization; the round-off error is 0.01. The function Ψ is described in text. In all cases the lower limit on x_1 is omitted as it is always confirmed.

Coalition Structure and corresponding Ψ-stable imputations	Game No.	Run No.	Observed Imputation Players 1	2	3	4	Incompatibilities between theory and data
$[\{1\}, \{2, 3, 4\}]$	1	1	.00	.40	.30	.30	$x_2 + x_3 = .70 < .75$
$x_2 + x_3 \geq .75$	1	2	.00	.43	.43	.15	None
$x_2 + x_4 \geq .50$	4	2	.00	.42	.42	.17	None
$x_3 + x_4 \geq .25$							
$x_1 = 0$							
$[\{2\}, \{1, 3, 4\}]$	4	1	.38	.00	.25	.38	None
$x_1 + x_3 \geq .50$							
$x_1 + x_4 \geq .25$	4	3	.29	.00	.46	.25	None
$x_3 + x_4 \geq .25$	4	8	.29	.00	.42	.29	None
$x_2 = 0$							
$[\{3\}, \{1, 2, 4\}]$	1	6	.43	.43	.00	.15	None
$x_1 + x_2 \geq .75$	1	8	.44	.44	.00	.11	None
$x_1 + x_4 \geq .25$	4	4	.38	.54	.00	.08	None
$x_2 + x_4 \geq .50$	4	5	.37	.53	.00	.10	None
$x_3 = 0$	4	7	.38	.54	.00	.08	None
$[\{1, 2, 3\}, \{4\}]$	1	4	.13	.44	.44	.00	Incompatible
$x_1 = .25$							
$x_2 = .50$	1	7	.19	.44	.38	.00	Incompatible
$x_3 = .25$							
$x_4 = .00$							
$[\{1, 4\}, \{2, 3\}]$	1	3	.13	.38	.38	.13	None
$x_1 + x_4 = .25$	4	6	.13	.38	.38	.13	None
$x_2 + x_3 = .75$							
$[\{1, 2\}, \{3, 4\}]$	1	5	.25	.50	.13	.13	None
$x_1 + x_2 = .75$							
$x_3 + x_4 = .25$							

conditions implied by Ψ-stability theory -- the first column -- condition ii has been omitted in all cases because it was always satisfied. In the last column it is indicated whether the data are compatible with the prediction. To be compatible, the data were only required to be within 0.01 of the predicted value since the reduction to normalized form introduced a round-off error. In the 32 cases there are four discrepancies; it is worth examining these in some detail.

In Table I, run 8 of game 3, resulted in the set of all players forming a single coalition and an equal split of the proceeds. For the given function Ψ, stability theory states that there is no stable pair with the set of all players as a coalition. On the other hand, the theory does admit the pair $(\|1/4,\ 1/4,\ 1/4,\ 1/4\|,\ [\{1\},\ \{2\},\ \{3\},\ \{4\}])$ as stable; however, experimentally, this case could not arise since there was no possible way for this coalition structure to collect a payment different from 0. So, had the players not wished to cooperate, they would have been forced by the experimental design to call themselves a single coalition in order to collect. Can it be that this explains this anomalous case?

The other three exceptions are in Table II, and they all occur in game 1 -- this was the S-equivalent form of the normalized non-symmetric game. In run 1, the error is five percentage points in a total predicted value of 75; by some standards this would not be considered a grevious error. In runs 4 and 7, the equilibrium coalition structure leads to a prediction of a unique imputation, and there can be no doubt that the subjects missed it. To be sure, there is some relation between the observed imputations and the predicted one, but there is no denying that they could not have calculated as the theory assumes they should.

§4. CONCLUSIONS

Excluding the peculiar case of the four-person coalition, the data are compatible with the predictions when the subjects received the characteristic function in normal form. The same is true for the S-equivalent form of the symmetric game, but not for the non-symmetric one. The incompatibilities in the last case tend to support the view that the participants did not fully appreciate the logic of the situation for a presentation of the characteristic function in a non-normalized form. I would argue, however, that by examining the data as we have done the evidence for non-equivalent treatment of S-equivalent games appears to be not nearly so strong as when average imputations are compared. One can be encouraged to hope that if the experiment were replicated and the 10 minute time limit eliminated, no exceptions would occur.

These remarks raise the question as to what a comparison of average imputations can mean; can such a comparison ever reject the assumption of S-equivalence? I would think not for the following reason: Any equilibrium theory so far constructed leads to multiple equilibrium imputations, and presumably the "correct" theory will do so also. However, such theories neither do nor can be expected to indicate the probability distribution over these equilibrium imputations. Presumably, a dynamic theory of coalition formation is necessary to predict the likelihood that a particular equilibrium state will be achieved. The data strongly suggest that these probabilities are dependent upon the form of presentation of the characteristic function, but it is much less certain that the form of presentation influences the states which are in equilibrium. This is to say, while two S-equivalent games may have corresponding equilibrium states, they need not have the same dynamic characteristics and, therefore, not the same probability distribution over equilibrium states. If this is so, then there is no reason at all to expect the average imputations of S-equivalent games to be the same, or even similar. This analysis encourages one not yet to reject the argument that equilibrium concepts should be invariant under S-equivalence; more data will have to be amassed before that is necessary.

Finally, the relative success of this analysis suggests that it may be neither completely foolish to postulate the existence of the "sociological" functions Ψ nor impossible to estimate them for certain experimental and existing situations.

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